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On flow of power-law fluids between adjacent surfaces: Why is it possible to derive a Reynolds-type equation for pressure-driven flow, but not for shear-driven flow?

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A R T I C L E I N F O

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A B S T R A C T

Flows of incompressible Navier–Stokes (Newtonian) fluids between adjacent surfaces are encountered in numerous practical applications, such as seal leakage and bearing lubrication. In seals, the flow is primarily pressure-driven, whereas, in bearings, the dominating driving force is due to shear. The governing Navier–Stokes system of equations can be significantly simplified due to the small distance between the surfaces compared to their size. From the simplified system, it is possible to derive a single lower-dimensional equation, known as the Reynolds equation, which describes the pressure field. Once the pressure field is computed, it can be used to determine the velocity field. This computational algorithm is much simpler to implement than a direct numerical solution of the Navier–Stokes equations and is therefore widely employed by engineers.

The primary objective of this article is to investigate the possibility of deriving a type of Reynolds equation also for non-Newtonian fluids, using the balance of linear momentum. By considering power-law fluids we demonstrate that it is not possible for shear-driven flows, whereas it is feasible for pressure-driven flows. Additionally, we demonstrate that in the full $3D$ model, a normal stress boundary condition at the inlet/outlet implies a Dirichlet condition for the pressure in the Reynolds equation associated with pressure-driven flow. Furthermore, we establish that a Dirichlet condition for the velocity at the inlet/outlet in the $3D$ model results in a Neumann condition for the pressure in the Reynolds equation.

1. Introduction

The fundamental governing equations of fluid flow are derived based on the principles of mass, momentum, energy balance and the second law of thermodynamics. When combined with appropriate boundary conditions and constitutive relations, these equations form a comprehensive set that describes the behavior of fluids under various flow conditions. However, this system of equations is highly complex and often poses numerical challenges, even with the aid of modern computers and software, in many realistic applications. Therefore, in engineering applications, it is essential to develop simplified mathematical models that offer computational efficiency, conceptual understanding of the flow, and suitability for optimal design, among other purposes. One area where successful derivation of such simplified models has been accomplished is when the fluid domain is thin, as observed in pipe flow, flow in Hele-Shaw cells, flow through porous media, and flow in channels between adjacent surfaces. In this article we consider shear-driven and pressure-driven flow of power-law fluids between adjacent curved surfaces. By a "shear-driven" flow, we refer to a flow that is engendered due to a boundary or boundaries of the flow domain being moved which leads to relative motion between the boundaries, while by "pressure-driven" flows we refer to flows in domains where the boundaries are fixed and the flow is caused due to application of a pressure gradient (see the definition of these flows in Section 2.4).

In the late 19th century, the British engineer Henry Selby Hele-Shaw conducted pioneering investigations into fluid flow through narrow gaps between parallel plates (Hele-Shaw, 1898). He conducted experiments using a transparent cell consisting of two closely spaced glass plates. One of Hele-Shaw's notable findings was the visualization of flow patterns and streamlines around various obstacles, such as cylinders, spheres, and plates. He observed phenomena such as flow separation, vortices, and the interaction between the fluid and the

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